QCD POTENTIAL MODEL FOR LIGHT-HEAVY QUARKONIA AND THE HEAVY QUARK EFFECTIVE THEORY

SURAJ N. GUPTA and JAMES M. JOHNSON*
Department of Physics, Wayne State University, Detroit, Michigan, 48202

ABSTRACT

We have investigated the spectra of light-heavy quarkonia with the use of a quantum-chromodynamic potential model which is similar to that used earlier for the heavy quarkonia. An essential feature of our treatment is the inclusion of the one-loop radiative corrections to the quark-antiquark potential, which contribute significantly to the spin-splittings among the quarkonium energy levels. Unlike $c\bar{c}$ and $b\bar{b}$, the potential for a light-heavy system has a complicated dependence on the light and heavy quark masses m and M, and it contains a spin-orbit mixing term. We have obtained excellent results for the observed energy levels of D^0, D_s, B^0, and B_s, and we are able to provide predicted results for many unobserved energy levels.

We have also used our investigation to test the accuracy of the heavy quark effective theory. We find that the heavy quark expansion yields generally good results for the B^0 and B_s energy levels provided that M^{-1} and $M^{-1}\ln M$ corrections are taken into account in the quark-antiquark interactions. It does not, however, provide equally good results for the energy levels of D^0 and D_s, which shows that the effective theory can be applied more accurately to the b quark than the c quark.

1. Introduction

The light-heavy quarkonia D, D_s, B, and B_s are at present of much experimental and theoretical interest, and their exploration is necessary for our understanding of the strong as well as the electroweak interactions. We shall here investigate the spectra of light-heavy quarkonia with the use of a quantum chromodynamic model similar to the highly successful model used earlier for the heavy quarkonia $c\bar{c}$ and $b\bar{b}$.[1] The complexity of the model is necessarily enhanced for a light-heavy system because the potential has a complicated dependence on the light and heavy quark masses m and M, and it contains a spin-orbit mixing term.

2. Light-heavy Quarkonium Spectra

Our treatment for the light-heavy quarkonia is similar to that for $c\bar{c}$ and $b\bar{b}$ except for the complications arising from the difference in the quark and antiquark masses.

*Presented by James M. Johnson

Thus, our model is based on the Hamiltonian

$$H = H_0 + V_p + V_c, \quad H_0 = (m^2 + \mathbf{p}^2)^{1/2} + (M^2 + \mathbf{p}^2)^{1/2} \tag{1}$$

where V_p and V_c are nonsingular quasistatic perturbative and confining potentials. Since our potentials are nonsingular, we are able to avoid the use of an illegitimate perturbative treatment.

The experimental and theoretical results for the energy levels of the light-heavy quarkonia D^0, B^0, D_s, and B_s, together with the ${}^3P_1'$-${}^1P_1'$ mixing angles arising from the spin-orbit mixing terms, are given in Table 1. For experimental data we have relied on the Particle Data Group[2] except that we have used the more recent results from the CLEO collaboration[3,4] for D_1^0, $D_2^{\star 0}$, and D_{s2} and from the CDF collaboration[5] for B_s. In this table, one set of theoretical results corresponds to the direct use of our model, while the other two sets are obtained by means of heavy quark expansions of our potentials to test the accuracy of the heavy quark effective theory with the inclusion of the M^{-1} and $M^{-1} \ln M$ corrections as well as without these corrections.

We expect the dynamics of a light-heavy system to be primarily dependent on the light quark. Therefore, our potential paramenters for D^0 and B^0 are the same except for the difference in the c and b quark masses. We have also ensured that the parameters for D_s and B_s are related to those for D^0 and B^0 through quantum chromodynamic transformation relations.

3. Conclusion

We have obtained excellent results for the observed energy levels of D^0, B^0, D_s, and B_s, and provided predicted results for many unobserved energy levels in Table 1. Although the use of a semirelativistic model may seem questionable for a system containing a light quark, ultimately such an approach should be judged on the basis of its predictions. Additional experimental data on the light-heavy quarkonia should be available in the near future.

We have also used our results to test the accuracy of the heavy quark effective theory. According to Table 1, the heavy quark expansion with the inclusion of the M^{-1} and $M^{-1} \ln M$ corrections yields generally good results for the B^0 and B_s energy levels. It does not, however, provide equally good results for the energy levels of D^0 and D_s, which indicates that the effective theory can be applied more accurately to the b quark than the c quark. We further find that the results for the energy levels in the limit $M \to \infty$ are unacceptable.

This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-85ER40209.

References

1. S. N. Gupta, J. M. Johnson, W. W. Repko, and C. J. Suchyta III, Phys. Rev D **49**, 1551 (1994). See also earlier related papers cited therein.
2. Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992).
3. CLEO Collaboration, Y. Kubota *et al.*, Phys. Rev. Lett. **72**, 1972 (1994).
4. CLEO Collaboration, P. Avery *et al.*, Phys. Lett. B **331**, 236 (1994).
5. CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **71**, 1685 (1993).

Table 1. D^0, D_s, B^0 and B_s energy levels in MeV. Effective theory results are given with the M^{-1} and $M^{-1}\ln M$ corrections as well as in the limit of $M \to \infty$.

	Expt.	Theory	Effective theory	$M \to \infty$
$1\,^1S_0$ (D^0)	1864.5±0.5	1864.5	1864.5	1864.5
$1\,^3S_1$ $(D^{\star 0})$	2007±1.4	2007.0	2010.9	1864.5
$2\,^1S_0$		2547.7	2566.5	2431.9
$2\,^3S_1$		2647.0	2662.1	2431.9
$1\,^3P_0$		2278.6	2310.2	2244.8
$1\,^3P_1'$		2407.3	2414.6	2244.8
$1\,^3P_2$ (D_2^0)	2465±4.2	2465.0	2474.0	2287.2
$1\,^1P_1'$ (D_1^0)	2421±2.8	2421.0	2438.2	2287.2
θ		29.0°	30.9°	35.6°
$1\,^1S_0$ (D_s)	1968.8±0.7	1968.8	1968.8	1968.8
$1\,^3S_1$ $(D_s^\star)$	2110.3±2.0	2110.5	2113.1	1968.8
$2\,^1S_0$		2656.5	2678.8	2536.5
$2\,^3S_1$		2757.8	2774.3	2536.5
$1\,^3P_0$		2387.8	2422.2	2382.2
$1\,^3P_1'$		2521.2	2528.8	2382.2
$1\,^3P_2$ (D_{s2})	2573.2±1.9	2573.1	2582.8	2402.8
$1\,^1P_1'$ (D_{s1})	2536.5±0.8	2536.5	2552.1	2402.8
θ		26.0°	31.8°	35.6°
$1\,^1S_0$ (B^0)	5278.7±2.1	5278.7	5278.7	5278.7
$1\,^3S_1$ $(B^{\star 0})$	5324.6±2.1	5324.0	5325.8	5278.7
$2\,^1S_0$		5892.1	5893.9	5846.3
$2\,^3S_1$		5924.3	5927.1	5846.3
$1\,^3P_0$		5689.5	5692.5	5659.1
$1\,^3P_1'$		5730.8	5734.1	5659.1
$1\,^3P_2$		5759.1	5761.4	5701.5
$1\,^1P_1'$		5743.6	5745.4	5701.5
θ		31.7°	31.3°	35.6°
$1\,^1S_0$ (B_s)	5383.3±6.7	5383.3	5383.3	5383.3
$1\,^3S_1$ $(B_s^\star)$	5430.5±2.6	5431.9	5434.1	5383.3
$2\,^1S_0$		6000.9	6003.1	5950.9
$2\,^3S_1$		6035.8	6039.1	5950.9
$1\,^3P_0$		5810.1	5814.2	5796.7
$1\,^3P_1'$		5855.0	5857.9	5796.7
$1\,^3P_2$		5875.2	5878.1	5817.1
$1\,^1P_1'$		5860.2	5863.2	5817.1
θ		27.3°	27.1°	35.6°